Report No. RR 47

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AN ANALYSIS OF THE DRAWBAR PULL VS SLIP RELATIONSHIP FOR TRACK LAYING VEHICLES

by

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Project No. 8010.11.822

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Authenticated:

U. S. Army Ordnance Tank-Automotive Command
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ABSTRACT

Equations are deduced by means of the Land Locomotion Soil Value System and a soil shear stress-strain relationship, first suggested in this paper, to predict the drawbar pull of track laying vehicles as a function of slippage. Test results indicating reasonable accuracy of the method are presented.

Conclusions are drawn concerning possible means of achieving improved tracked vehicle design, and the direction of future research.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS
(Continued)

FOREWORD

SUMMARY:

This report describes the derivation of a new group of equations to predict the drawbar pull-slip relationships of a track laying vehicle by means of the Land Locomotion Soil Value System. This report also describes the test procedure and results performed with small scale models to check the accuracy of the new equations. The soil values were first measured for each particular soil prior to running the models. Two electrically driven models, each of the same weight, but with different track dimensions, were used for the tests. The track speed and the distance traveled by the model were measured for the slip calculations and the constant weight on the gravity dynamometer was measured for the drawbar pull.

FINDINGS:

The test results, shown in graphical form in figures 6-16, indicate that reasonable accuracy can be obtained to predict the drawbar pull of a model vehicle using the equations presented. The band, or spread, of the predicted curves were determined from the maximum and minimum measured soil strength properties at the ground contact pressures of the models.

CONCLUSIONS:

It is believed that the correlation of the measured and predicted data illustrates the usefulness of an analysis based on empirical soil-stress-strain relationships. Although the pressure distribution under the loaded area was idealized, it is anticipated that with further study more complicated pressure distributions have possibilities of being analyzed.

RECOMMENDATIONS:

It is recommended that tests be conducted to determine the sinkage-slip relationship in several soils so that further refinements can be obtained in the area of soil-vehicle relationships. Studies should also be conducted on pressure distribution under a loaded area that are not idealized so that the Land Locomotion Soil Value System can be more rigorously applied to vehicles with all types of vehicle geometry.

KEY TO SYMBOLS AND DEFINITIONS

H	Traction, tractive effort, thrust. The gross tractive effort developed by the vehicle...(lbs)
i_o	Slip, slippage. A ratio between the difference of the theoretical and actual speed and the theoretical speed.
s	Shear stress in the soil...(lbs/in^2)
s_{max}	Maximum shear stress in the soil...(lbs/in^2)
C	Cohesion...(lbs/in^2)
P	Pressure normal to the shear-plane...(lbs/in^2)
P_{max}	Maximum normal pressure under a track which occurs at Z_o sinkage...(lbs/in^2)
ϕ	Angle of shear resistance, the angle of internal friction...(Degrees)
A	Vehicle-ground interface, ground contact area...(in^2)
W	Load...(lbs)
K_1	Slippage coefficient...(in^{-1})
K_2	Slippage coefficient
Y_{max}	Maximum value of terms included in the brackets in Equation 3.
x	Horizontal distance between an arbitrarily chosen point on the track and the leading point of the interface...(in)
ℓ	Length of the track...(in)
j	Soil deformation in the horizontal direction...(in)
K	Deformation modulus of a soil shear stress-strain curve...(in)
b	Width of one track...(in)
h	Heignth of grousers...(in)

DP	Drawbar pull, net tractive effort. The thrust lessened by the motion resistance...(lbs)
R	Motion resistance...(lbs)
Z	Sinkage at a point defined by x...(in)
Z_o	Maximum sinkage of a track, sinkage at the end of the interface...(in)
k_c	Cohesive modulus of sinkage...(lbs/in^{n+1})
k_ϕ	Frictional modulus of sinkage...(lbs/in^{n+2})
n	Exponent of sinkage
m	Number of terms considered in the series in equation 20
J	Traction exponent, abbreviation for $\frac{i_o \ell}{K}$
T	Torque needed to turn the shearhead of the Bevameter...(in lbs)
r_i	Inside radius of the shearhead (annulus)...(in)
r_o	Outside radius of the shearhead (annulus)...(in)
r	Radius of shearhead associated with dF...(in)
dF	Elemental area r dr dθ...(in^2)
dθ	Angle at center of annulus associated with dA...
$\bar{S}_{max}$	Arithmetical average of measured S_{max} values...(psi)
g	Regression coefficient
$\bar{p}$	Arithmetical average of applied p values...(psi)
i	Number S_{max} and p values considered in the regression analysis

PROJECT TITLE: AN ANALYSIS OF THE DRAWBAR PULL VS. SLIP RELATIONSHIP FOR TRACK LAYING VEHICLES

BACKGROUND:

Micklethwaite[1] was the first to attempt to relate soil shearing strength as used in soil mechanics to determine the available thrust for a given soil. Starting with Coulombs well know equation[2]

$$s_{max} = c + p \tan \phi \tag{1}$$

he multiplied both sides by the ground contact area

$$H_{max} = s_{max}A = cA + pA \tan \phi.$$

For uniform normal pressure distribution the relationship between weight and pressure is: W = pA. Thus Micklethwaite's equation took the following form:

$$H_{max} = cA + W \tan \phi \tag{2}$$

Bekker[3] suggested a more general equation which has been successfully used by the Land Locomotion Laboratory to predict track laying vehicle performance in snow[4]. Bekker made use of the fact that the form of soil shear stress-strain curves are similar to the shape of the displacement-natural frequency diagram of an aperiodic damped vibration[5]. (See Figure 1.)

By replacing the damping constant and the spring constant by appropriate soil parameters he arrived at the following equation:

$$S = \frac{c + p \tan \phi}{Y_{max}} \left[e^{(-K_2 + \sqrt{K_2^2 - 1})K_1 i_o x} - e^{(-K_2 - \sqrt{K_2^2 - 1})K_1 i_o x} \right] \tag{3}$$

The shear stress can be integrated along the track-soil interface. For constant normal pressure distribution, Bekker derived the following equation.

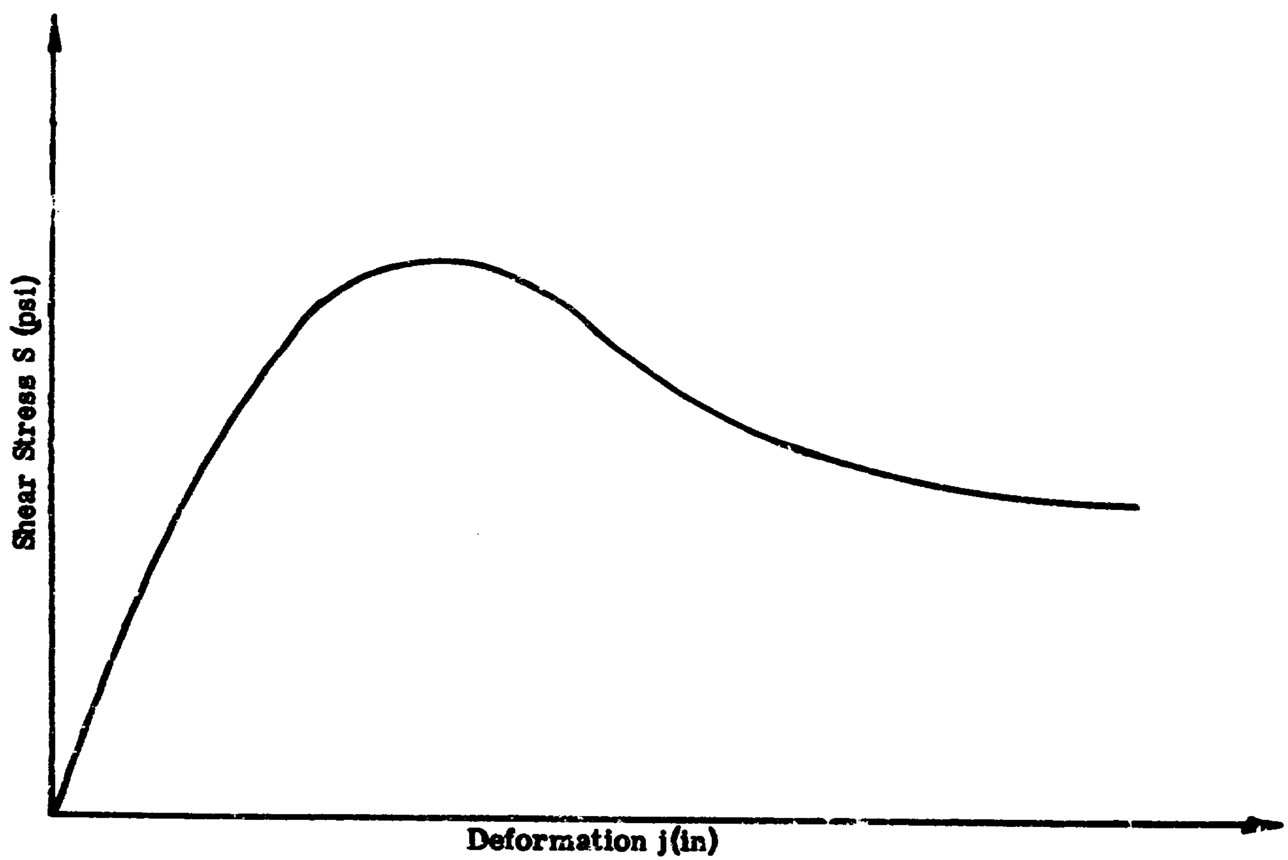

FIGURE 1 SOIL SHEAR STRESS-STRAIN CURVE WITH A "HUMP"

$$n = \frac{2\ell b(c + p\tan\phi)}{K_1 i_o y_{max}\ell}\left[-\frac{1}{-K_2+\sqrt{K_2^2-1}} + \frac{e^{(-K_2+\sqrt{K_2^2-1})K_1 i_o \ell}}{-K_2+\sqrt{K_2^2-1}} + \frac{1}{-K_2-\sqrt{K_2^2-1}} - \frac{e^{(-K_2-\sqrt{K_2^2-1})K_1 i_o}}{-K_2-\sqrt{K_2^2-1}}\right] \qquad (4)$$

For more complicated pressure distribution, geometrical solutions developed by Weiss in 1956 have been applied[6]. It has been found at the Land Locomotion Laboratory that in the majority of the cases the soil does not exhibit a hump and a decay in the shear stress-strain curve (figure 2). Thus it is seldom necessary to use an involved equation such as equation 3. Also, it has been difficult to establish the numerical value of K_2 when the curve does not drop after reaching its maximum. These observations lead the authors to suggest the following simpler equation to describe soil shear stress-strain curves of the type shown in figure 2:

$$S = (c + p\tan\phi)\,(1 - e^{-j/K}) \qquad (5)$$

Note that for very large deformation equation 5 approaches Coulomb's formula:

$$(s \rightarrow c + p\tan\phi) \text{ as } j \rightarrow \infty$$

Differentiating both sides of equation 5 and setting j equal to zero the following is obtained:

$$\left(\frac{ds}{dj}\right)_{j=0} = \frac{c + p\tan\phi}{K} \qquad (6)$$

FIGURE 2. TYPICAL SOIL SHEAR STRESS-STRAIN CURVE

Equation 6 represents the slope of the tangent drawn at the origin (figure 3), therefore K may be obtained from the experimental shear stress-strain curve as the distance between the intercept of the tangent drawn at the origin and the line given by $s = c + p \tan \phi$ and the s axis. K is called the deformation modulus of a soil shear stress-strain curve.

DISCUSSION OF THE THEORY:

The total tractive effort exhibited by a track is obtained by integrating the shear stress along the ground contact area. Thus from equation 5 assuming constant width (b):

$$H = 2b \int_0^{\ell} (c + p \tan \phi) \; (1 - e^{-j/K}) dx \qquad (7)$$

There are two variables other than x in the integrand: p and j. Hence equation 7 can only be evaluated if the function $p = p(x)$ and $j = j(x)$ are known.

Uniform Normal Stress Distribution.

1. Traction. The case of uniform pressure distribution, p = constant, is considered first. The soil deformation under a track is proportional to the distance of the soil particle from the front end of the soil-vehicle interface (x). The factor of proportionality is defined as slip (i_o). Thus:

$$j = i_o x$$

Hence equation 7 may be rewritten in the following form:

$$H = 2b(c + p \tan \phi) \int_0^{\ell} (1 - e^{\frac{-i_o x}{K}}) dx \qquad (8)$$

The integration yields:

$$H = 2b \left[c + p \tan \phi \right] \left[\ell + \frac{K}{i_o} (e^{\frac{-i_o \ell}{K}} - 1) \right] \qquad (9)$$

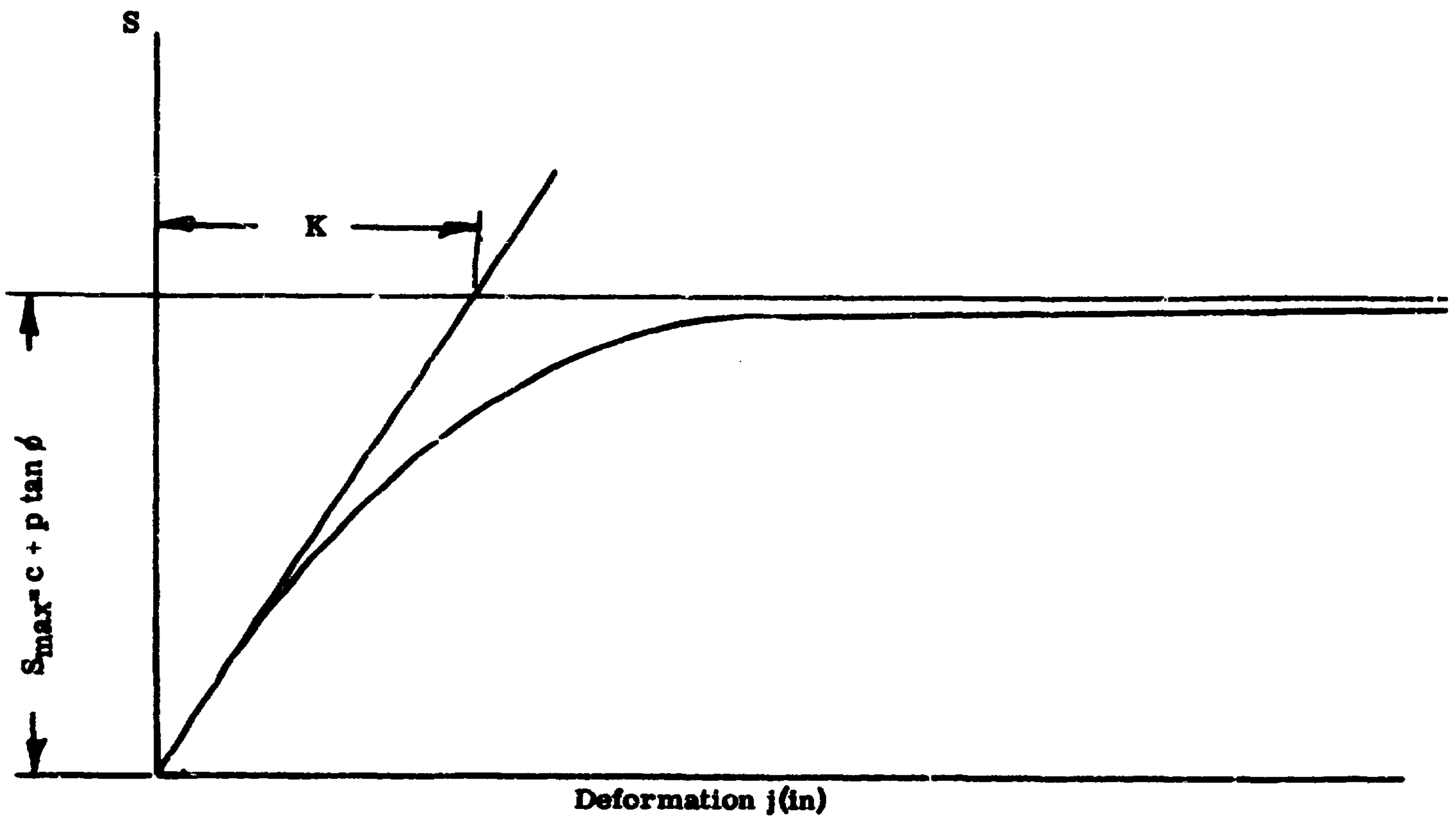

FIGURE 3. EVALUATION OF "K" FROM A TYPICAL SOIL SHEAR STRESS-STRAIN CURVE

Using the following notations:

$$A = 2b\ell, \quad W = pA \text{ and } J = \frac{i_o \ell}{K}$$

equation 9 becomes:

$$H = (Ac + W \tan \phi) \left[1 - \frac{1}{J} \quad (1-e^{-J}) \right] \qquad (10)$$

Thus Micklethwait's solution (equation 2) always yields too high values except for infinite length. However the error committed will be insignificant if J is a large number.

2. Effect of Grousers. In case a track is equipped with grousers the maximum thrust is somewhat higher than that predicted by equation 10. The additional thrust may be calculated by means of an equation established by Bekker[3]. Accordingly equation 2 becomes:

$$H = Ac \left(1 + \frac{2h}{b}\right) + W \tan \phi \left\{ 1 + 0.64 \left[\frac{h}{b} \cot^{-1}\left(\frac{h}{b}\right) \right] \right\}$$

Thus a refined form of equation 10 may be established as shown below:

$$H = \left[Ac \left(1 + \frac{2h}{b}\right) + W \tan \phi \left\{ 1 + 0.64 \left[\frac{h}{b} \cot^{-1} \left(\frac{h}{b}\right) \right] \right\} \right] \left[1 - \frac{1}{J} \left(1 - e^{-J}\right) \right] \qquad (11)$$

It has been assumed that the shear stress may be expressed with the same relationship in both vertical and horizontal planes.

3. Drawbar Pull. In order to obtain the drawbar pull one has to subtract the motion resistance from the traction:

$$DP = H - R \qquad (12)$$

The total resistance consists of several components. Only the resistance due to soil compaction is considered here because its magnitude is dominating in most cases.

According to Bekker[3] the compaction resistance may be expressed as follows:

$$R_c = 2 \frac{k_c + bk_\phi}{n+1} Z_o^{\,n+1} \qquad (13)$$

where:

$$Z_o = \left[\frac{W}{A\left(\frac{k_c}{b} + k_\phi\right)} \right]^{\frac{1}{n}} \qquad (14)$$

It should be emphasized that a relationship which would enable one to consider the sinkage as a function of slippage is lacking as yet. Equations 13 and 14 are accurate for small slippages only.

Linear Sinkage.

1. Traction. If the sinkage is linearly proportional with x the solution is somewhat more involved. The vertical pressure under a footing as a function of the sinkage may be expressed by an equation first introduced by Bekker[7]:

$$p = \left(\frac{k_c}{b} + k_\phi\right) Z^n \qquad (15)$$

From Figure 4:

$$Z = Z_o \frac{x}{\ell} \qquad (16)$$

Using equations 15 and 16, equation 5 now becomes:

$$S = \left[c + \left(\frac{k_c}{b} + k_\phi\right)\left(\frac{Z_o}{\ell}\right)^n x^n \tan\phi \right] \left(1 - e^{\frac{-i_o x}{K}}\right) \qquad (17)$$

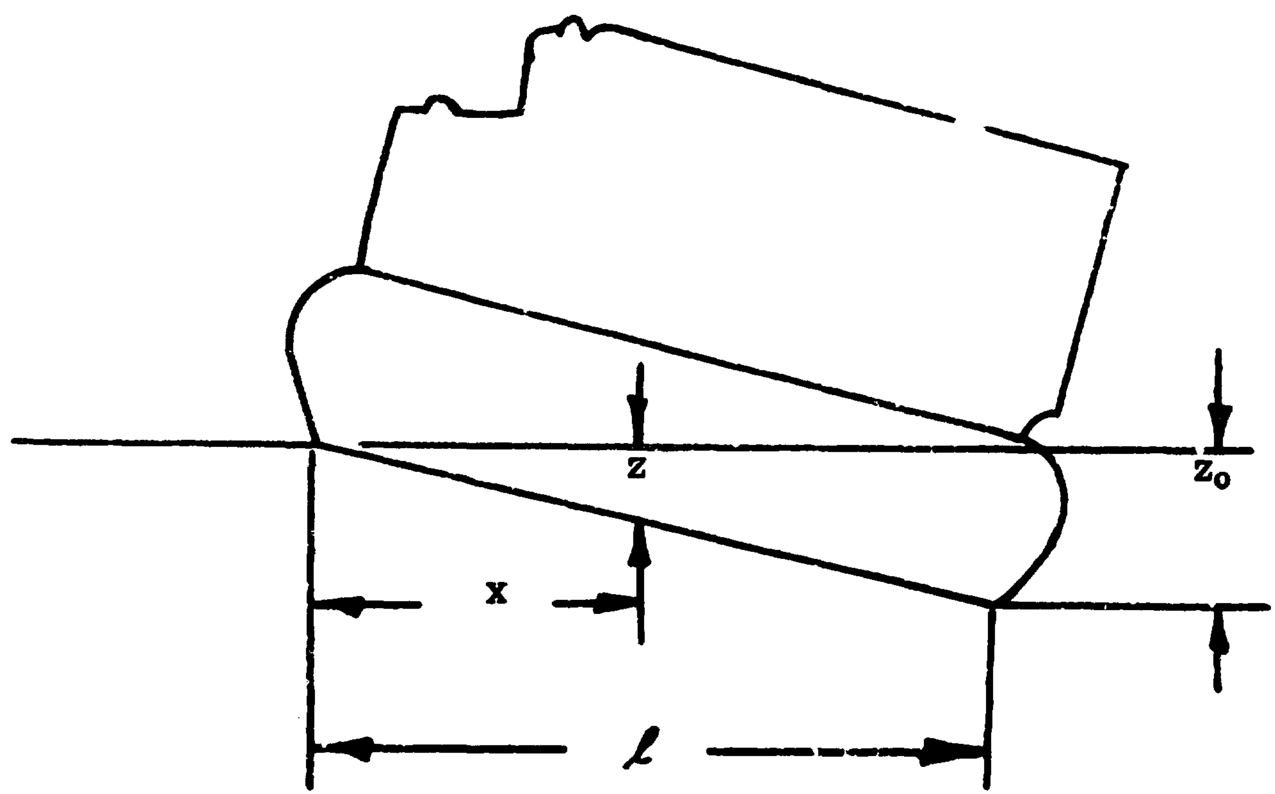

FIGURE 4. TRACKED VEHICLE IN "TRIMMED" POSITION

and,

$$H = 2b \int_0^{\ell} \left[c + \left(\frac{k_c}{b} + k_\phi\right) \left(\frac{Z_o}{\ell}\right)^n x^n \tan \phi \right] \left(1-e^{\frac{-i_o x}{K}}\right) dx \quad (18)$$

Equation 18 reduces to the following form:

$$H = A \left\{ c \left[1 - \frac{1}{J} \left(1-e^{-J}\right) \right] + p_{max} \tan \phi \left[\frac{1}{n+1} - \frac{1}{\ell^{n+1}} \int_0^{\ell} x^n e^{\frac{-i_o x}{K}} dx \right] \right\} \quad (19)$$

Here $p_{max} = \left(\frac{k_c}{b} + k_\phi\right) Z_o^n$, and Z_o is given by equation 25.

The integral in the last term can only be solved in a closed form if n is an integer. Numerical solutions presented in this paper have been obtained by means of a Datatron 204 Electronic Computer.

When a digital computer is not available the solution may be obtained by means of power series. It can be shown that this method yields the following equation:

$$H = A \left\{ c \left[1 - \frac{1}{J} \left(1-e^{-J}\right) \right] + p_{max} \tan \phi \sum_{m=1}^{\infty} \frac{J^m (-1)^{m+1}}{m!(m+n+1)} \right\} \quad (20)$$

Since the number of terms required in the series may be quite large, a simpler approximative solution is now presented.

If $\frac{i_o}{K}$ is not a very small number and ℓ is a large number the following approximation may be applied:

$$\int_0^{\ell} x^n e^{\frac{-i_o x}{K}} \cong \int_0^{\infty} x^n e^{\frac{-i_o x}{K}} dx$$

The right hand side can be rewritten as:

$$\frac{1}{\left(\frac{i_o}{K}\right)^{n+1}} \int_0^{\infty} \frac{i_o}{K} \quad x^n \, e^{\frac{-i_o x}{K}} \, d\left(\frac{i_o x}{K}\right)$$

The integrand is the so called Gamma function[8] of n + 1. Thus:

$$\frac{1}{\frac{i_o}{K}^{n+1}} \int_0^{\infty} \frac{i_o}{K} \, x^n \, e^{\frac{-i_o x}{K}} \, d\left(\frac{i_o x}{K}\right) = \frac{\Gamma(n+1)}{\left(\frac{i_o}{K}\right)^{n+1}} \tag{21}$$

It is known that $\Gamma(n+1) = n!$

Thus when using equation 21 the total tractive effort becomes:

$$H = A \left\{ c \left[1 - \frac{1}{J}\left(1 - e^{-J}\right) \right] + p_{max} \tan \phi \left[\frac{1}{n+1} - \frac{n!}{J^{n+1}} \right] \right\} \tag{22}$$

2. Grouser Effect. The effect of grousers may be included again. The solution is then (see equation 19).

$$H = A \left\{ c \left(1 + \frac{2h}{b}\right) \left[1 - \frac{1}{J} \left(1 - e^{-J}\right) \right] + \right.$$

$$p_{max} \tan \phi \left[1 + 0.64 \left(\frac{h}{b} \cot^{-1} \frac{h}{b}\right) \right] \left[\frac{1}{n+1} - \frac{1}{\ell^{n+1}} \right.$$

$$\left. \left. \int_0^{\ell} x^n e^{\frac{-i_o x}{K}} dx \right] \right\} \tag{23}$$

and the approximate solution is:

$$H = A \left\{ c \left(1 + \frac{2h}{b}\right) \left[1 - \frac{1}{J} \left(1 - e^{-J}\right) \right] + \right.$$

$$\left. p_{max} \tan \phi \left[1 + 0.64 \left(\frac{h}{b} \cot^{-1} \frac{h}{b}\right) \right] \left[\frac{1}{n+1} - \frac{n!}{J^{n+1}} \right] \right\} \tag{24}$$

3. Drawbar Pull. It has been shown[6] that the maximum sinkage may be calculated by means of the following equation:

$$z_o = \left[\frac{(n+1)\,W}{A\left(\frac{k_c}{b} + k_\phi\right)} \right]^{\frac{1}{n}} \tag{25}$$

and the compaction resistance is again:

$$R_c = \frac{\left(\frac{k_c}{b} + k_\phi\right) z_o^{\,n+1}}{n+1} \tag{26}$$

Where z_o is given by equation 25.

The drawbar pull of the vehicle may be found as:

$$DP = H - R_c$$

where H is given by equation 23 or 24 and R_c by equation 26.

TESTS:

In order to check the accuracy of the theory presented above, numerous full scale vehicle and model tests have been carried out by the Land Locomotion Laboratory in 1960. These tests are reported in another report. Here only the results of a series of scale model tests conducted by the authors are presented.

Test Apparatus.

Powered small scale models were run in different soils with varying loads on the tow hook and the slip was measured. The loading system was a gravity dynamometer (figure 5). The drive shaft of the model was fitted with a hexagon nut to trip a micro-switch which actuated a counter resulting in a trace on the recording paper of a brush recorder. This enabled one to compute the theoretical distance covered by the vehicle. A potentiometer connected to the load dynamometer pulley measured the actual distance traveled.

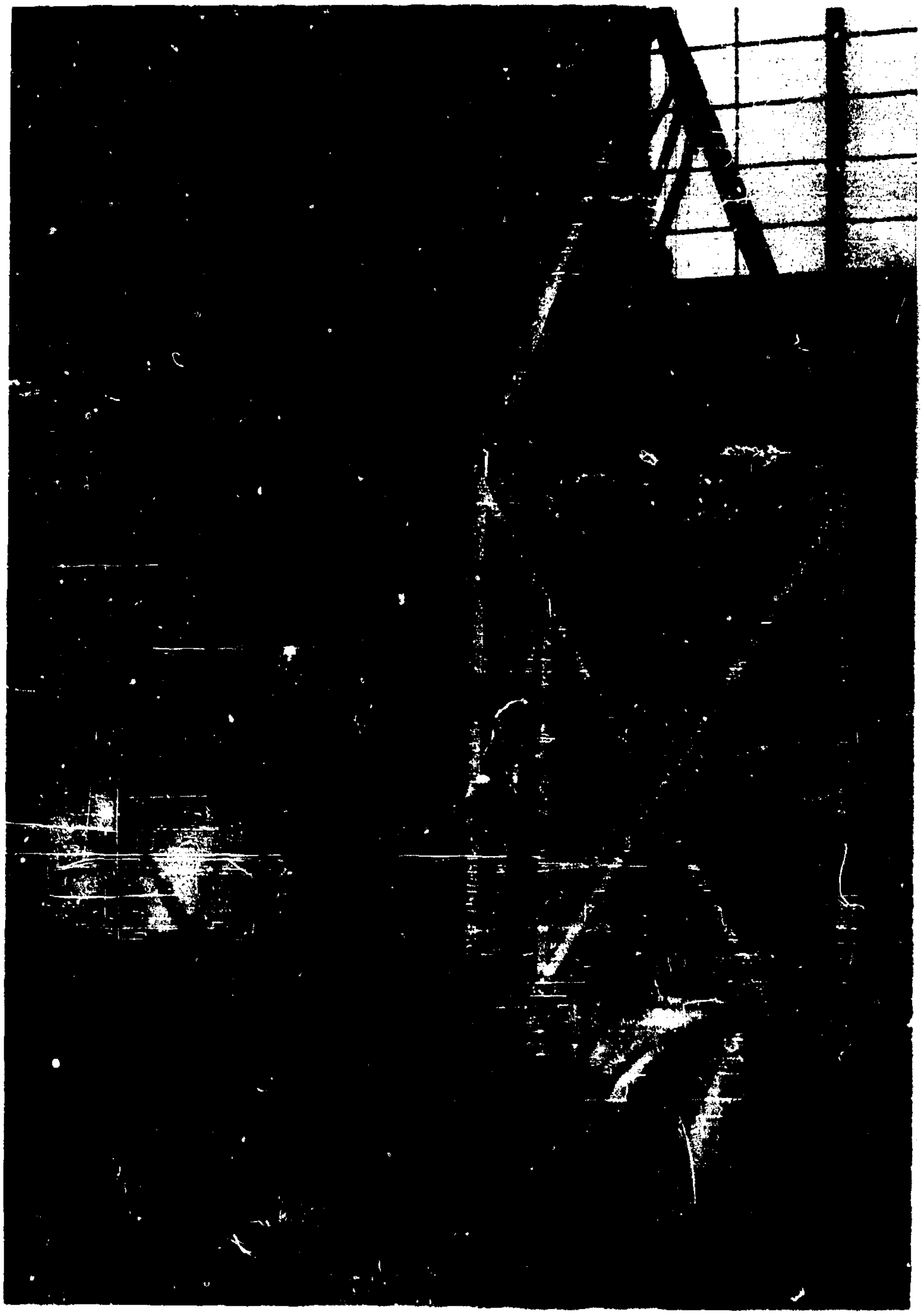

FIGURE 5. MODEL DP VS SLIP TEST (SCALE MODEL TEST BIN WITH GRAVITY DYNAMOMETER)

Model "R" weighed 162 lbs. and its tracks had the dimensions 17.1 x 2.375 inches; Model "B" weighed 162 lbs. and had tracks with the dimensions 17.1 x 1.875 inches. The height of the grousers was 5/32 inches for both models.

Test Procedure.

1. The soil was processed by a rotary tiller and either rolled with a lawn roller or tamped with an air temper depending upon the moisture of the soil.

2. Bevameter soil measurements[9] were taken at random locations throughout the length of the soil bin. Two sets of load-sinkage and shear stress-strain measurements were taken.

3. Moisture content samples were taken at each end and in the middle of the bin.

4. Step 1 repeated.

5. The model was placed in the bin and started. The load on the dynamometer pan was increased in ten pound increments until the vehicle stalled in order to determine the range of loads to be tested.

6. The model was moved six inches from the ruts of the previous run. Each following run was completed with one of the loads within the established range. Thus the entire slip range was covered with at least 4 different loads, each repeated 3 times.

Evaluation of Test Data.

It was realized that the problem of keeping the soil consistant for each set of runs would have been difficult. The moisture content samples indicated that there were differences along the length of the bin for any one run. To encompass the performance of the models within the soil variations, each set of shear curves was analyzed separately. A linear regression analysis was performed with each set of data[11]. The straight line of Coulomb's shear stress equation for the four sets of data taken, were plotted and the maximum and minimum shear stress at the ground pressure of the model were determined. The corresponding c and ϕ values were used to determine the maximum and minimum drawbar pull predictions. Thus instead of predicting a single drawbar pull vs. sinkage curve, a "band" was established. The predicted and measured values are presented in figures 6 through 16. A sample of the computations is presented in the appendix.

CONCLUSIONS:

1. It is believed that correlation of tested data obtained by the theory presented demonstrates the usefulness of an analysis based on empirical soil stress-strain relationships.

2. The equations presented are accurate if road wheels are close to each other, thus forming a "solid footing". In case of excessive track slack and more involved pressure distribution, a geometrical method to describe track behavior seems to be the most promising.

3. Further research is needed to establish the sinkage-slippage relationship.

4. An additional conclusion based on an analysis of the equations derived is that for a given ground contact area (or mean ground pressure) the longer and narrower track is advantageous. Since, however, a too long track encounters difficulties in steering, the concept of articulated vehicle arises as a possible avenue toward the improvement of present design trends, as first emphasized by Bekker[10].

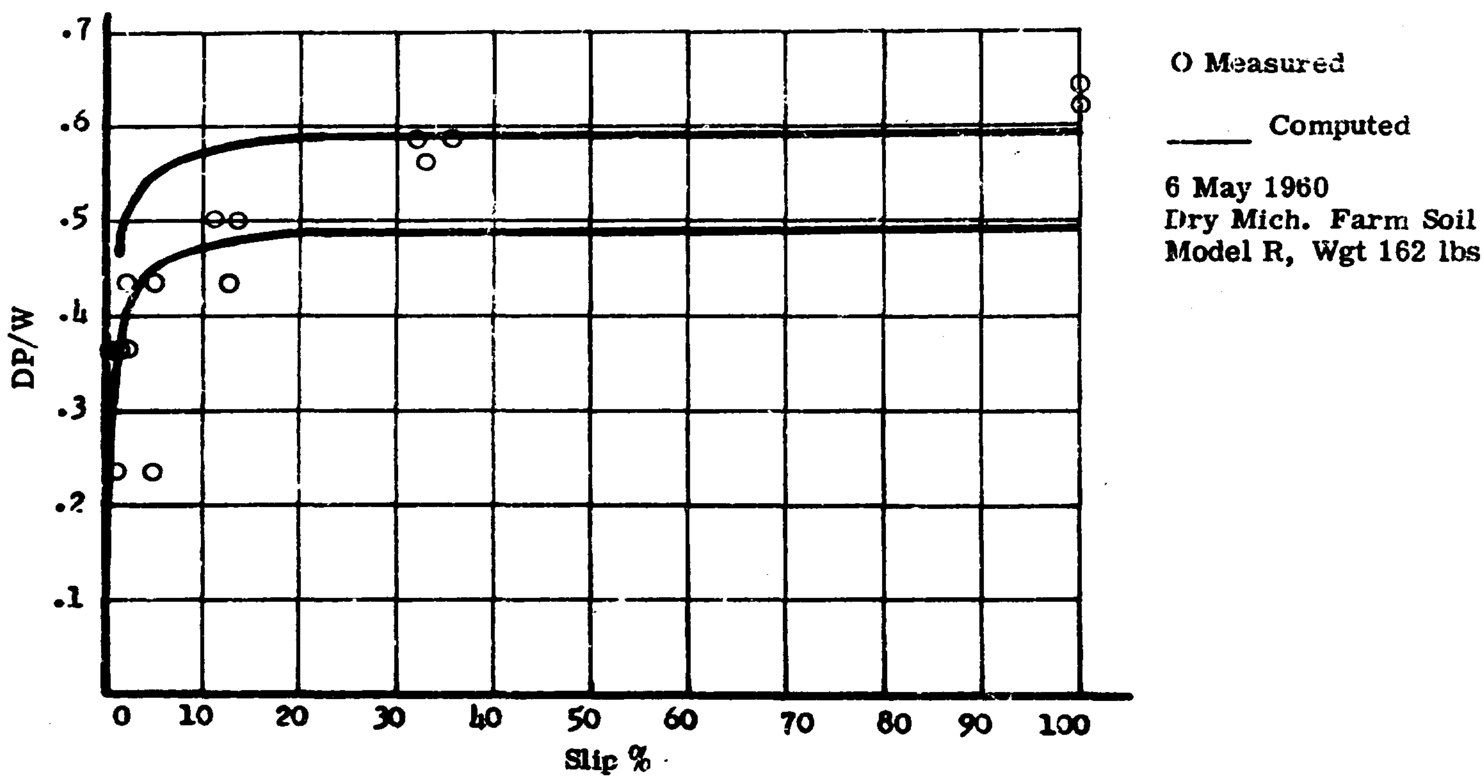

FIGURE 6. PREDICTED "BAND" AND MEASURED VALUES OF DRAWBAR PULL PER UNIT WEIGHT AS A FUNCTION OF SLIP FOR MODEL "R" IN FARM SOIL

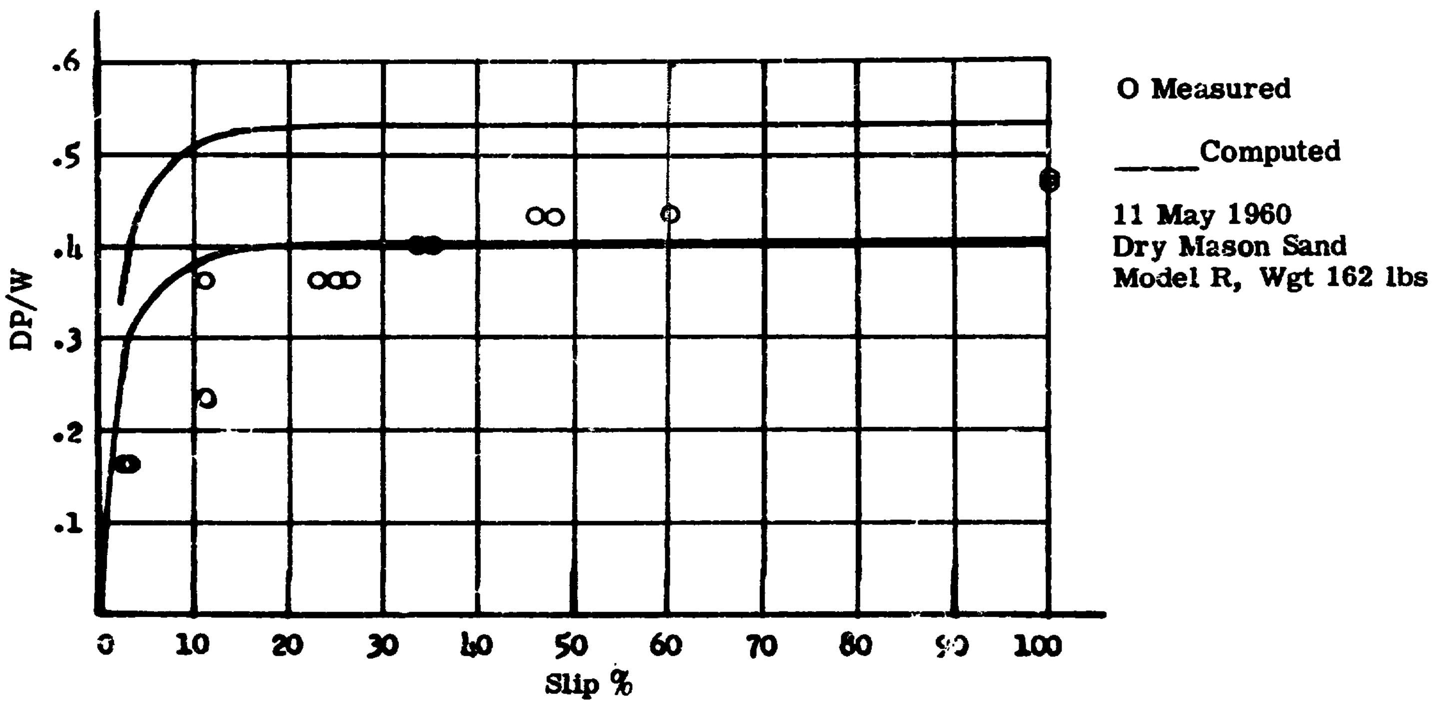

FIGURE 7. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "R" IN DRY MASON SAND

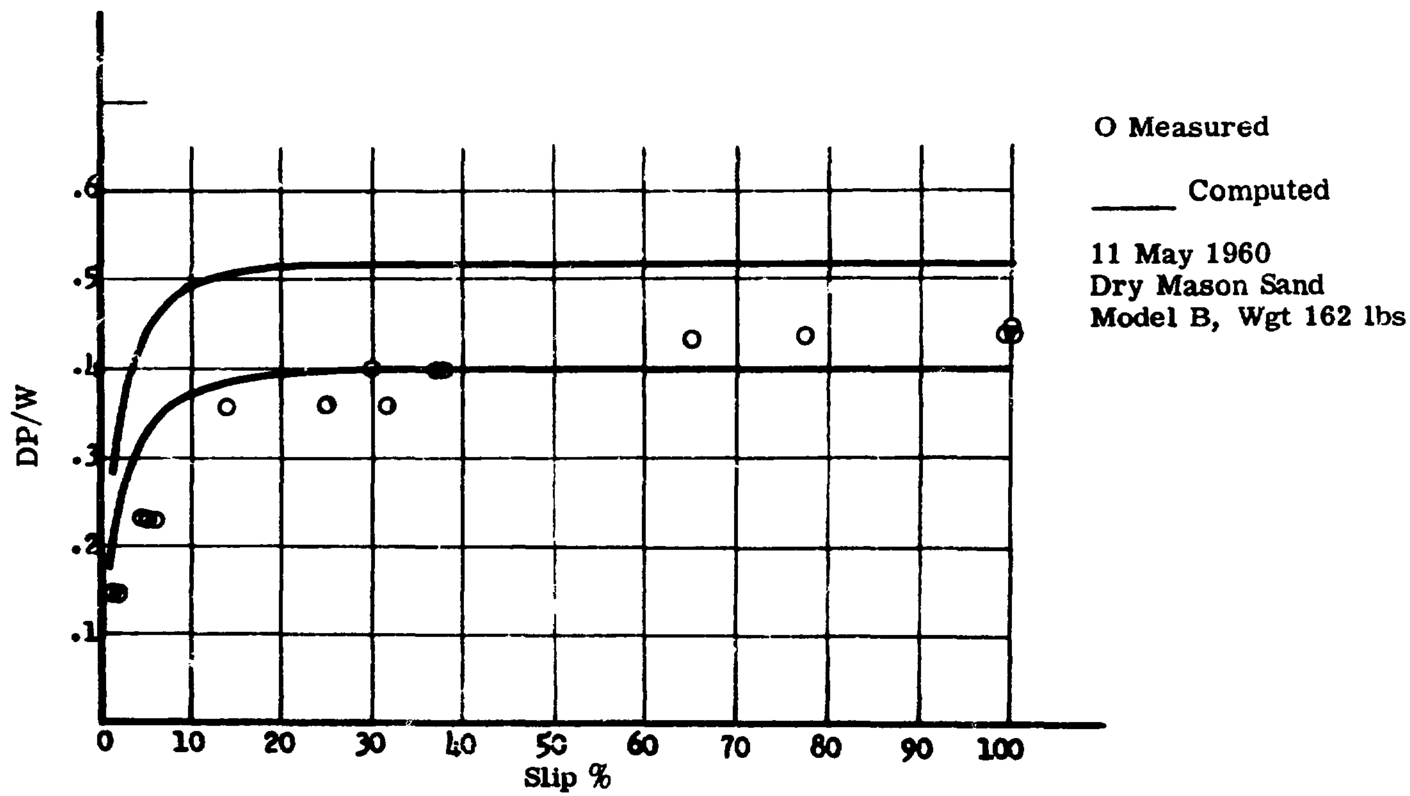

FIGURE 8. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "B" IN DRY MASON SAND

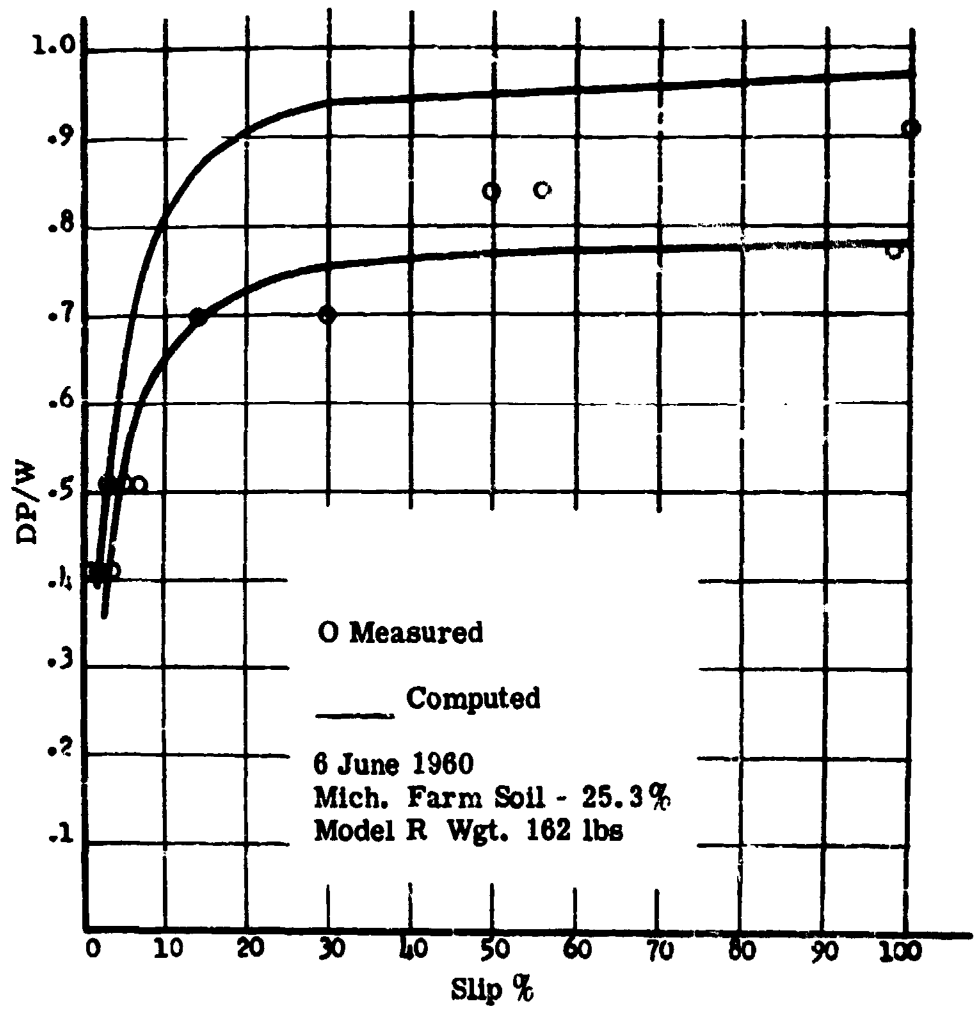

FIGURE 9. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "R" IN FARM SOIL OF MC = 25%

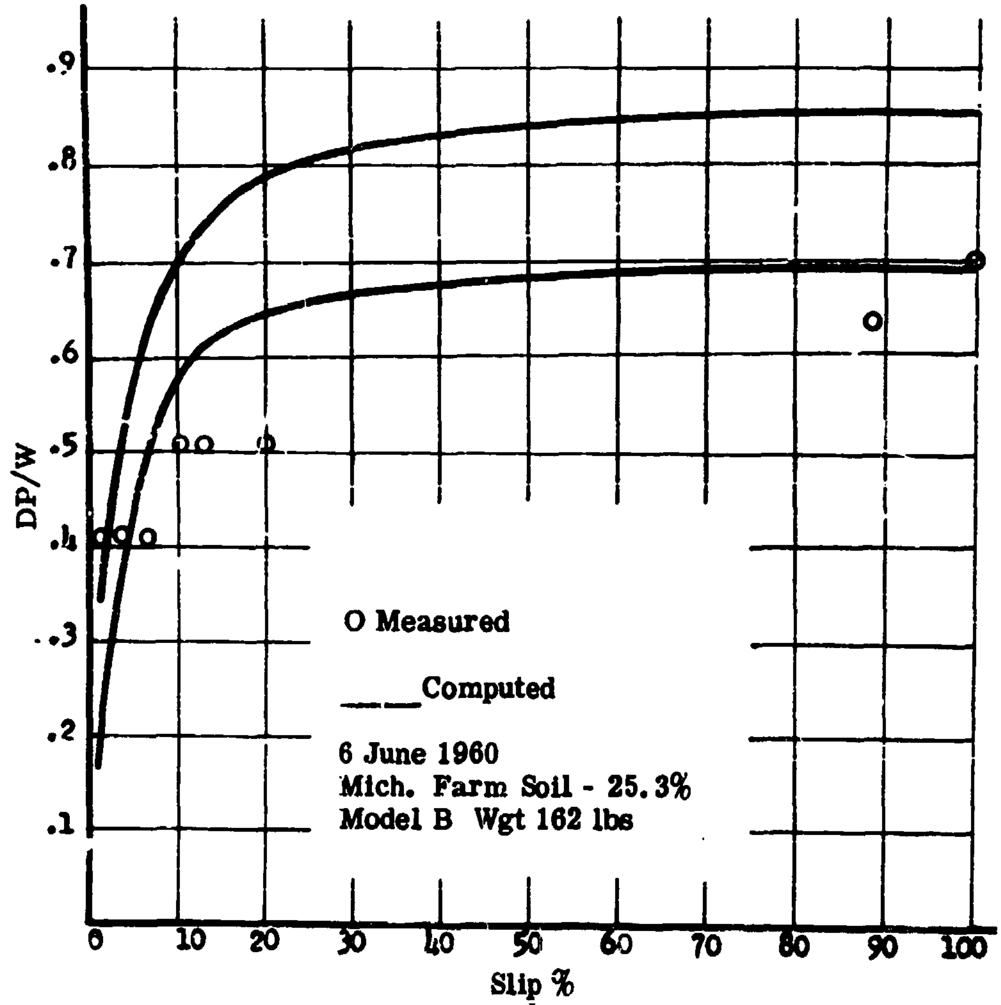

FIGURE 10. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "B" IN FARM SOIL OF MC = 25%

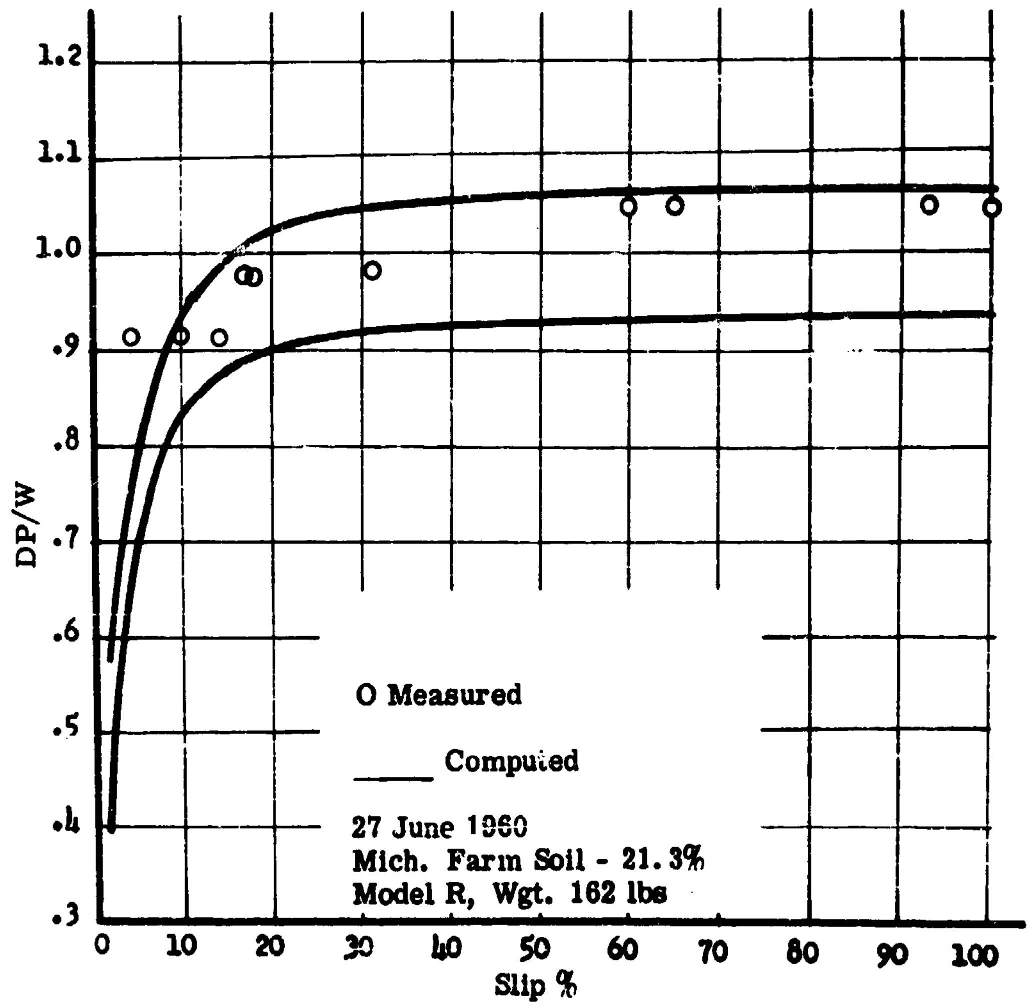

FIGURE 11. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "R" IN FARM SOIL OF MC = 21.3%

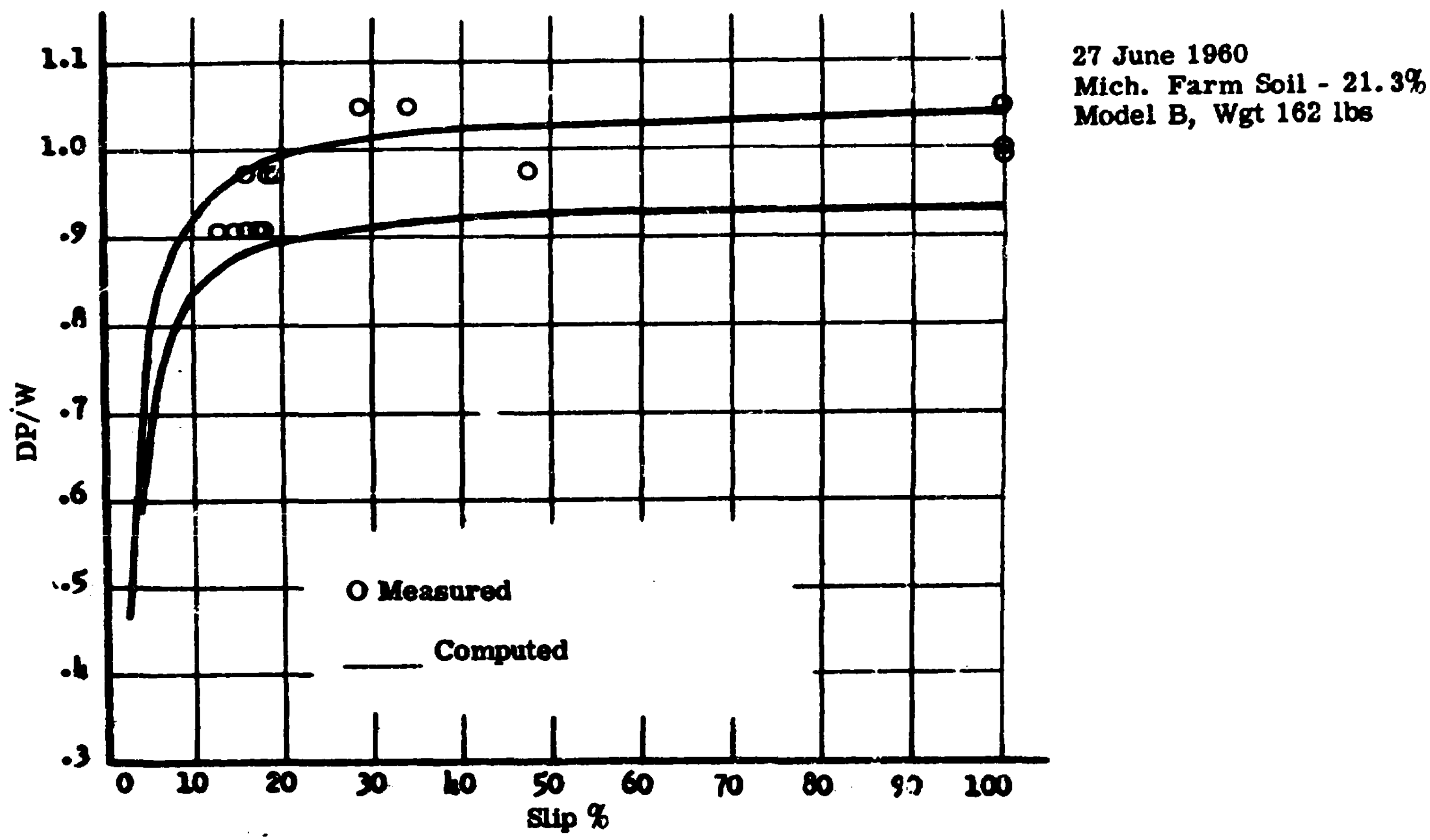

FIGURE 12. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "B" IN FARM SOIL OF MC = 21.3%

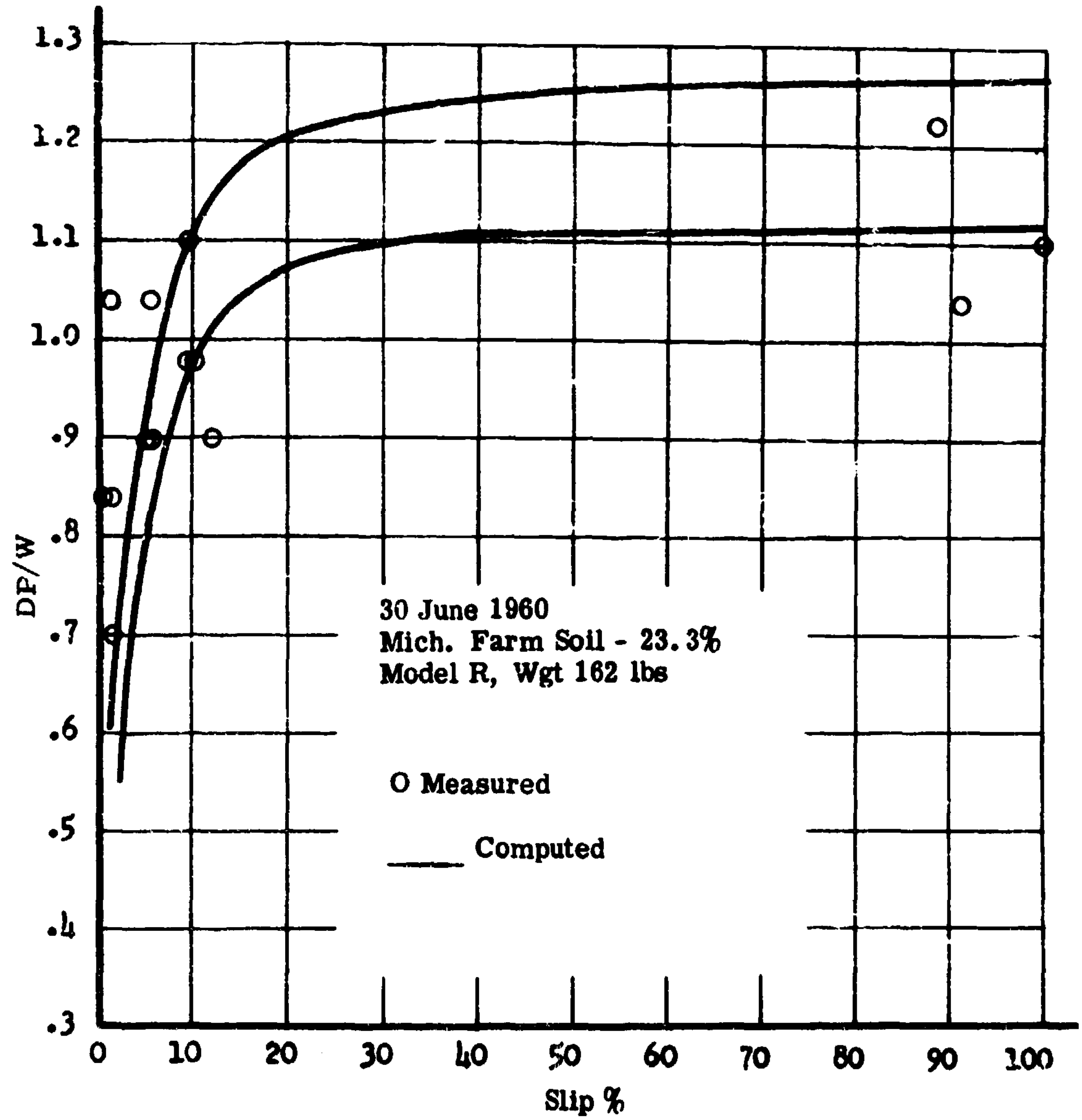

FIGURE 13. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "R" IN FARM SOIL OF MC = 23.3%

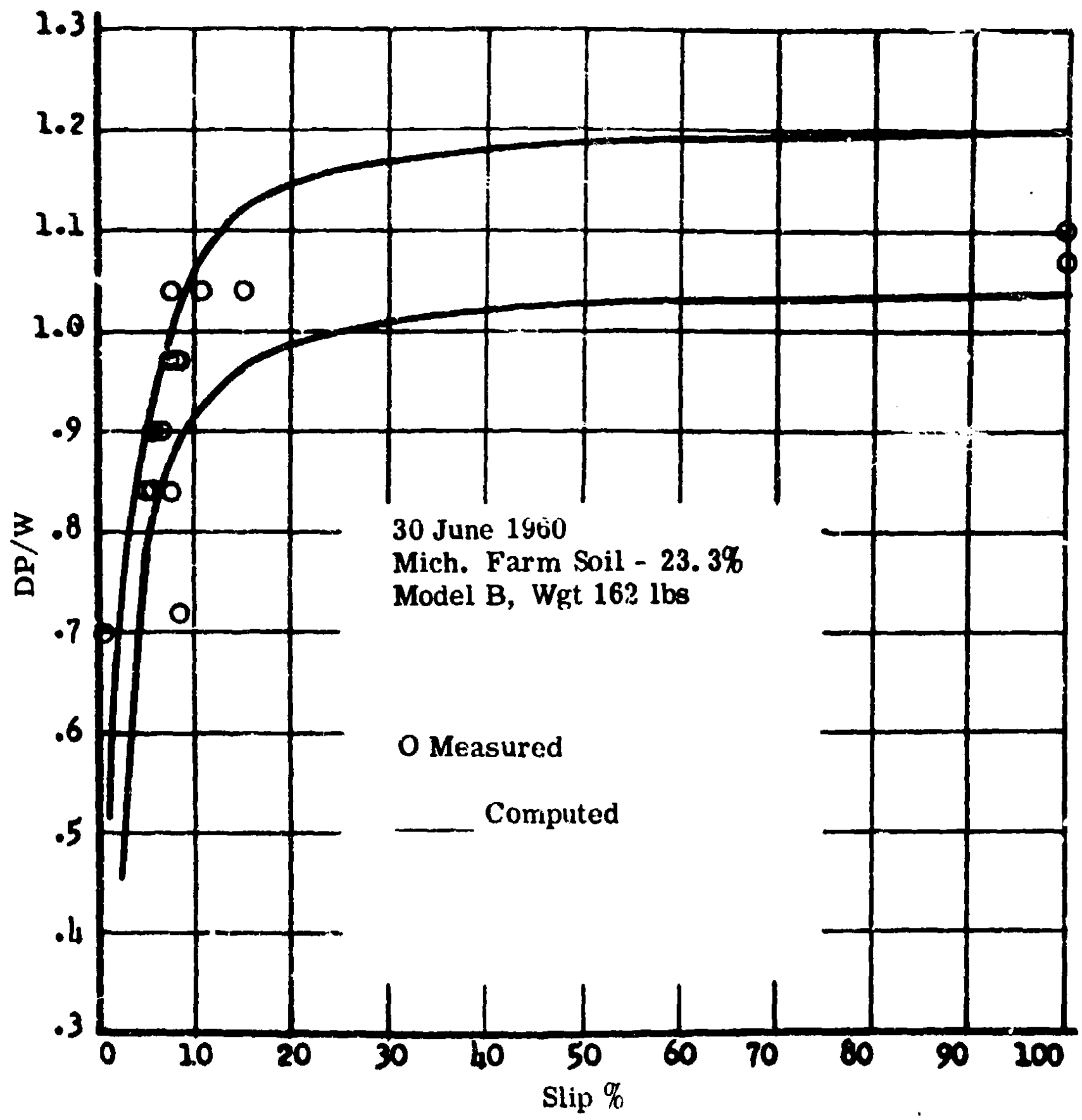

FIGURE 14. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "B" IN FARM SOIL OF MC = 23.3%

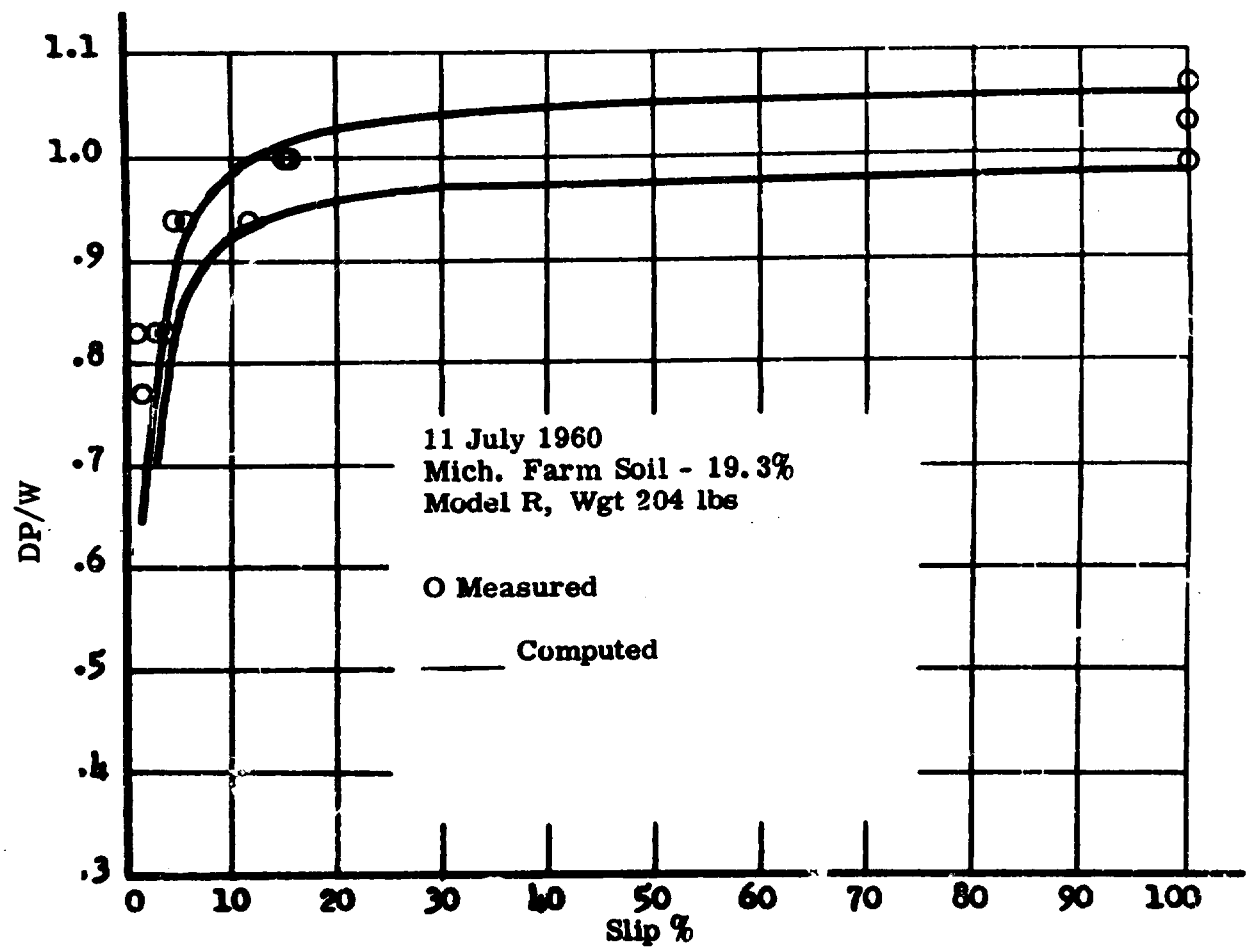

FIGURE 15. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "R" IN FARM SOIL OF MC = 19.3%

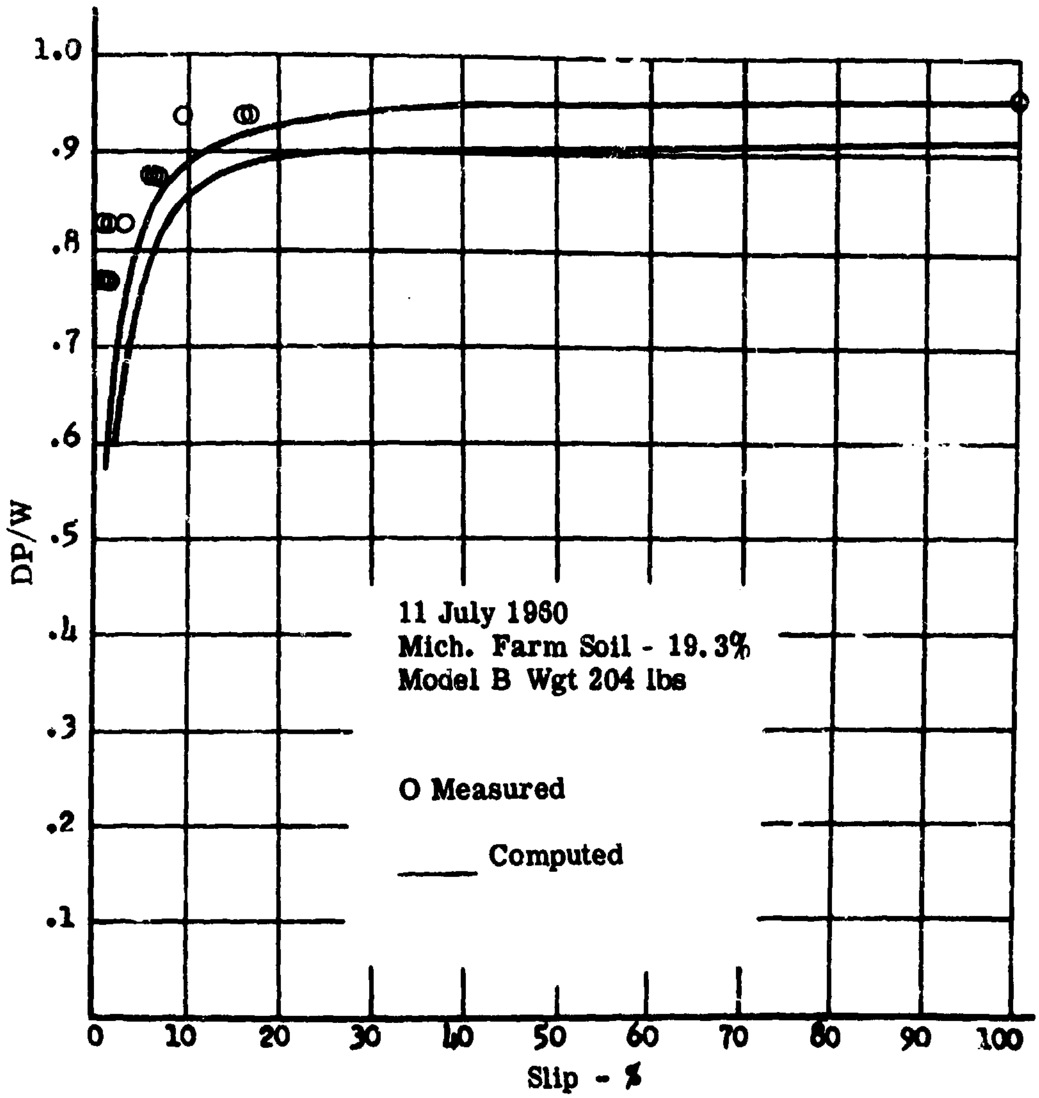

FIGURE 16. PREDICTED AND MEASURED DP/W VALUES AS A FUNCTION OF SLIP FOR MODEL "B" IN FARM SOIL OF MC = 19.3%

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APPENDIX

On 27 June 1960 the following soil moisture data was measured at three different locations in the test bin.

TABLE 1. Soil Moisture Data at 8:30 A.M.

	North End	Middle	South End
Weight of Sample (gr)	107.2	131.5	119.8
Weight of Dry Soil Sample	88.7	107.9	98.2
Weight of Water (gr)	18.5	23.6	21.6
Water Content %	20.85	21.87	20.99

Here the water content is the following:

$$WC\% = \frac{\text{Weight of water}}{\text{Weight of dry soil sample}} \; 100$$

TABLE 2. Soil Moisture Data at 3:30 P.M.

	North End	Middle	South End
Weight of Sample (gr)	102.0	127.4	131.0
Weight of Dry Soil Sample (gr)	84.7	104.8	107.1
Weight of Water	17.3	22.6	23.9
Water Content	20.42	21.56	22.32

The mathematical average of the six water content data is 21.3%

Shear Test Evaluation

Date: 27 June 1960

Soil: Michigan Farm Soil

Water Content: 21.3

Shear tests were performed by means of a Bevameter[9]. The following equations were used to evaluate the shear stress:

$$\int dT = S\int_{r_1}^{r_o}\int_0^{2\pi} r\, dF = S\int_{r_1}^{r_o}\int_0^{2\pi} r^2 dr d\theta$$

Hence:

$$S = \frac{3T}{2\pi(r_o^3 - r_1^3)}$$

since, for the annulus used the dimensions were: $r_o = 3.50$ in. and $r_1 = 2.75$ in (yielding an area of $F = 14.72\ in^2$)

$S = 0.02158T$

The unloaded shearhead weighed $W_e = 13.5$ lbs

TABLE 3. Shear Test Data.

Location & Time	Load Applied on Shear-head W (lbs)	Pressure $p = W + W_e/F$ (psi)	Measured Torque T(in/lbs)	Max. Shear Stress S_{max} (psi)	Deformation Modulus K (in)
South Half of bin at 8:30 A.M.	21.5	1.46	63	1.36	0.36
	29.5	2.00	87	1.88	0.29
	37.5	2.54	113	2.44	0.31
	45.5	3.09	126	2.72	0.36
	53.5	3.63	148	3.19	0.42
North Half of bin at 8:30 A.M.	21.5	1.46	73	1.58	0.22
	29.5	2.00	92	1.99	0.29
	37.5	2.54	113	2.44	0.42
	45.5	3.09	137	2.96	0.31
	53.5	3.63	155	3.34	0.22

TABLE 3. Shear Test Data
(Continued)

	Load Applied on Shear-head W (lbs)	Pressure $p = W + W_e/F$ (psi)	Measur-ed Torque T(in/lbs)	Max. Shear Stress S_{max} (psi)	Deforma-tion Modulus K (in)
South Half of bin at 3:00 P.M.	21.5	1.46	63	1.36	0.33
	29.5	2.00	83	1.79	0.29
	37.5	2.54	103	2.22	0.31
	45.5	3.09	122	2.63	0.33
	53.5	3.63	150	3.24	0.33
North Half of bin at 3:00 P.M.	21.5	1.46	68	1.47	0.36
	29.5	2.00	82	1.80	0.31
	37.5	2.54	107	2.31	0.33
	45.5	3.09	129	2.78	0.36
	53.5	3.63	153	3.30	0.31

K was determined from the shear-stress strain curve, S, (see figure 3).

Linear Regression Analysis

In order to determine the best fitting straight line for $S_{max} = \mathcal{f}(p)$ (see equation 1) to the measured shear stress data the following equation was used:

$$S_{max} = \bar{S}_{max} + g(p - \bar{p})$$

Here

$$g = \frac{\sum pS_{max} - \frac{\sum p \sum S_{max}}{i}}{\sum p^2 - \frac{(\sum p)^2}{i}}$$

Figures 17 and 18 were obtained using p and S_{max} values as listed in table 3 and equation 27.

The average ground pressure for model "B" was 2.40 psi for which the smallest S_{max} value was 2.1 psi and the largest S_{max} was 2.33 (see figures 17 and 18).

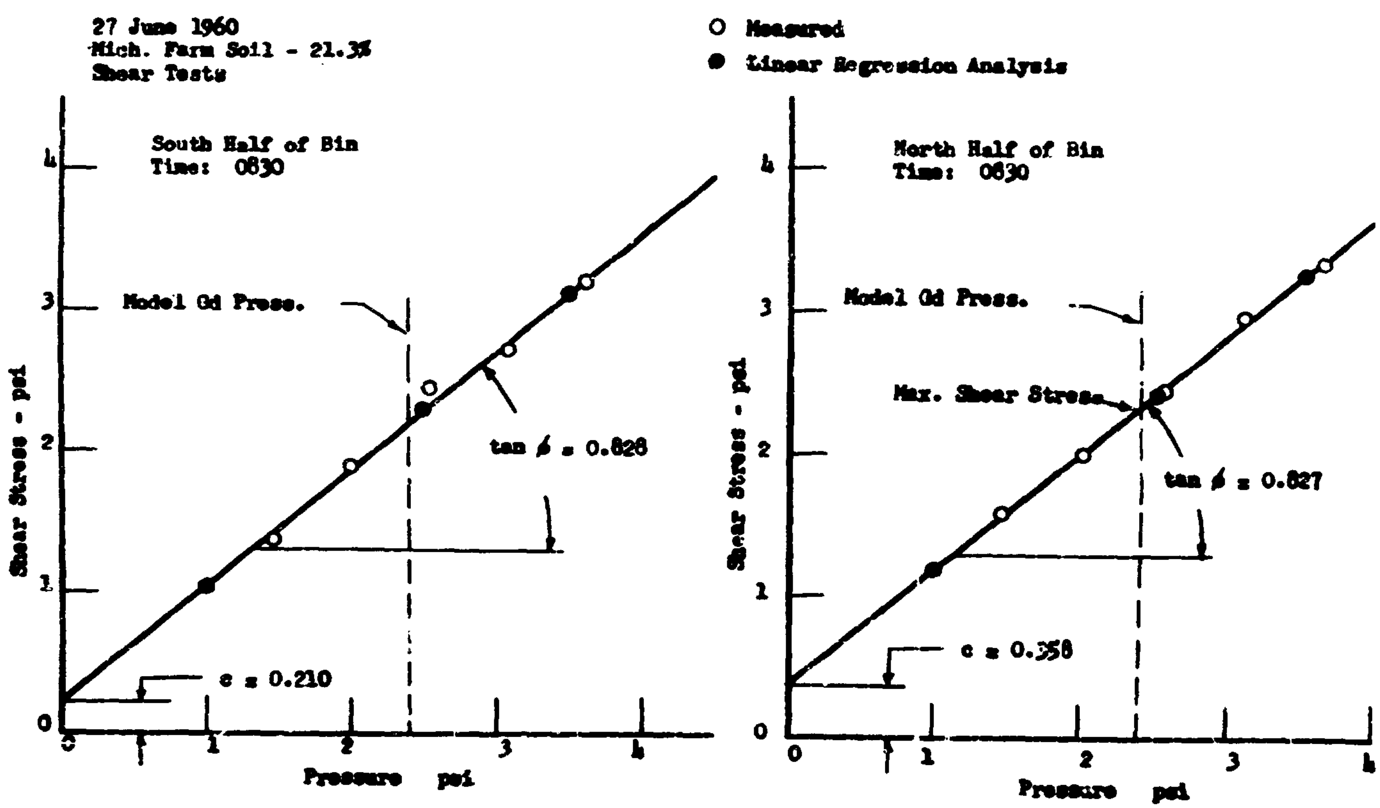

FIGURE 17. EVALUATION OF "C" AND Ø FROM A MAXIMUM SHEAR STRESS VS. NORMAL PRESSURE CURVE

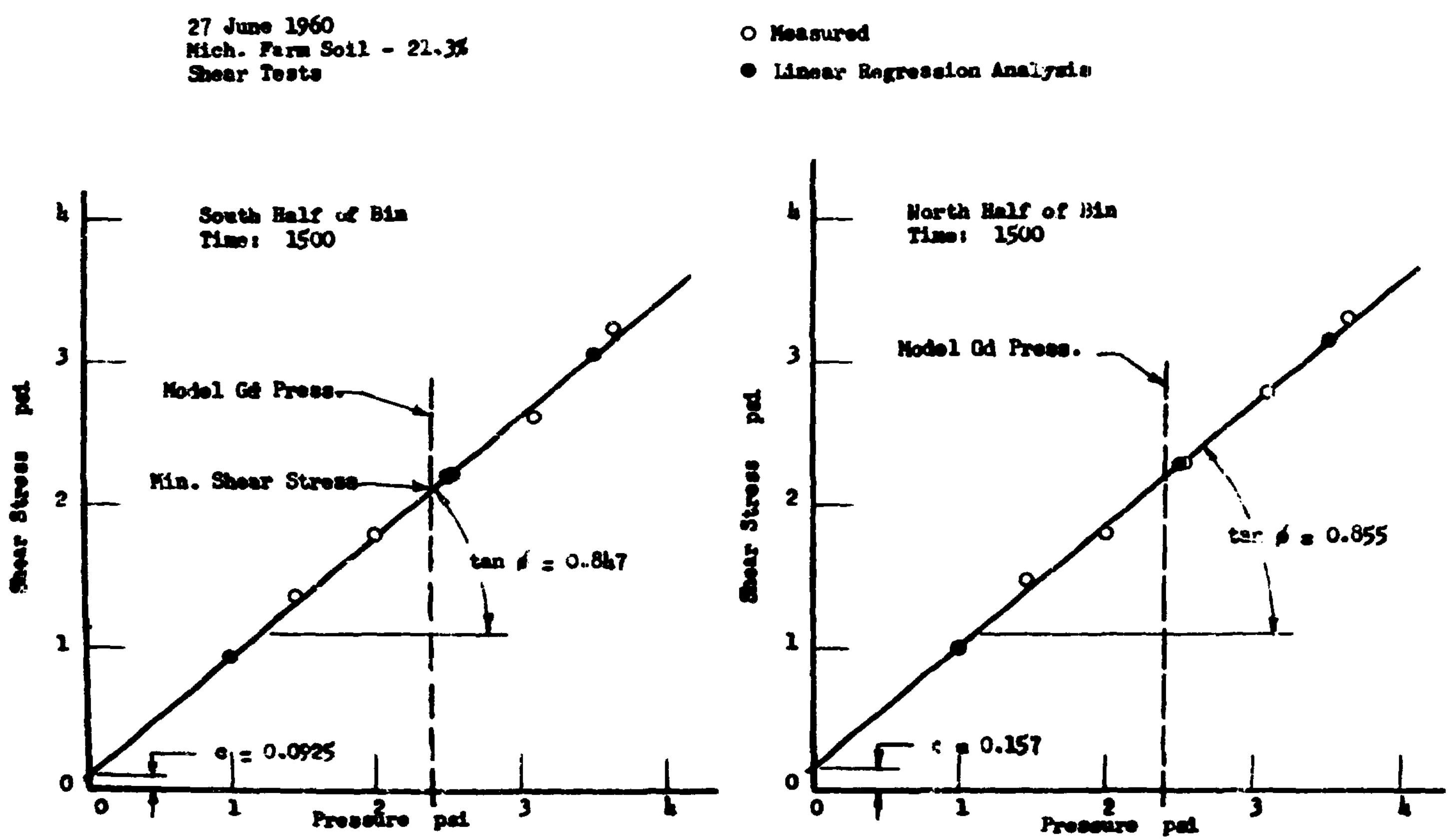

FIGURE 18. EVALUATION OF "C" AND "Ø" FROM A MINIMUM SHEAR STRESS VS. NORMAL PRESSURE CURVE

The corresponding c and ϕ values as determined from figures 17 and 18 were c = 0.0925 psi, $\phi = \tan^{-1} 0.847$ (rad) and c = 0.358 (psi) $\phi = \tan^{-1} 0.827$ (rad) respectively. (Note that the minimum S_{max} is not necessarily associated with the minimum c or ϕ values.)

These c and ϕ values were used in equation 23 to obtain the upper and lower drawbar pull vs slip curves in figures 6-16.

For the evaluation K was assumed to be 0.30 (in) which corresponds to the average ground pressure of model "B". (See table 3.)

In order to compute

$$P_{max} = \left(\frac{k_c}{b} + k_\phi\right) Z_o^{\,n} \quad \text{and } R_c \text{ (equation 26)} k_c,$$

k_ϕ and n were needed. Since small sinkages of the order of 0.25 in. were observed during the test runs the effect of k_c, k_ϕ and n on the drawbar pull was assumed negligible and therefore a regression analysis similar to that described above was not performed on the measured data.

The arithmetical average for four penetration tests performance on 27 June 1960 $k_c = 3.5$; $k_\phi = 3.5$ and n = 0.40 were used in equation 26 to obtain figures 6 through 16.

27 JUNE 1960
MICH. FARM SOIL 21.3%
MODEL WEIGHT - 162 LBS.

DRAWBAR PULL - SLIP TESTS

SHEET NO.	RUN NO.	PAN WGT. LBS.	TRUE WGT. LBS.	LINEAR DISPL. mm	ACTUAL DIST. in.	SPROC. COUNT	THEO DIST. IN.
1	1	131	148	15.0	50.6	47	53.1
1	2	141	158	15.0	50.6	55	61.9
		151	170	STALL			
		151	170	STALL			
1	3	141	158	6.0	21.3	27	30.4
2	1	141	158	13.2	44.8	58	65.2
2	2	151	170	12.7	43.2	108	121.5
2	3	131	148	14.2	48.3	50	56.3
1	4	131	148	15.3	51.6	51	57.4
2	4	151	170	3.6	12.8	96	108.0
		151	170	STALL			
2	5	151	170	16.0	53.5	136	153.0

SHEET NO.	RUN NO.	PAPER DISPL. mm	TIME sec.	SPEED FT/MIN	SLIP %	DP/W
1	1	101.5	10.2	24.8	4.7	.913
1	2	111.0	11.1	22.8	18.3	.976
					100.0	1.05
					100.0	1.05
1	3	174.7	17.5	6.1	17.2	.976
2	1	108.1	10.8	20.8	31.2	.976
2	2	225.6	22.6	9.6	60.0	1.05
2	3	96.0	9.6	25.2	14.1	.913
1	4	96.0	9.6	26.9	10.1	.913
2	4	183.1	18.3	3.5	93.0	1.05
					100.0	1.05
2	5	241.3	24.1	11.1	65.0	1.05

27 JUNE 1960
MICH. FARM SOIL - 21.3%
MODEL WEIGHT 162 LBS.

DRAWBAR PULL - SLIP PREDICTION

SOIL VALUES

$k_c = 3.5$

$k_\phi = 3.3$

$n = 0.40$

$c \begin{cases} \min = 0.0925 \\ \max = 0.3582 \end{cases}$

$\tan\Phi \begin{cases} \min = 0.8473 \\ \max = 0.8270 \end{cases}$

$K = 0.30$

MODEL VALUES

$\ell = 18.0$ IN.

$b = 2.375$ IN.

GROUSER HEIGHT $h = 5/32''$

CORRECTION FOR GROUSER AFFECT

$$H = cA\left(1 + \frac{2h}{b}\right) + W \tan\Phi \left\{1 + .64\left[\frac{h}{b}\cot^{-1}\frac{h}{b}\right]\right\}$$

$$H = cA\left(1 + \frac{.3124}{2.375}\right) + W \tan\Phi \left\{1 + .64\left[\frac{.1562}{2.375}\cot^{-1} .06577\right]\right\}$$

$$H = 1.1315\ cA + 1.063\ W \tan\Phi$$

CORRECTED VALUES

$c \begin{cases} \text{MIN.} = 0.1047 \\ \text{MAX.} = 0.4053 \end{cases}$

$\tan\Phi \begin{cases} \text{MIN.} = 0.9007 \\ \text{MAX.} = 0.8791 \end{cases}$

$$\frac{k_c}{b} + k_\phi = \frac{3.5}{2.375} + 3.3 = 4.77$$

$$A = 2b\ell = 2(2.375)\,(18) = 85.5 \text{ IN}^2$$

$$Z_o = \left[\frac{W(n+1)}{2b\ell\left(\frac{k_c}{b} + k_\phi\right)}\right]^{\frac{1}{n}} = \left[\frac{162(1.4)}{855(4.77)}\right]^{2.5} = \left[.556\right]^{2.5} = 0.23 \text{ IN.}$$

$$R_c = 2\left[\frac{b\left(\frac{k_c}{b} + k_\phi\right)}{n+1}\right](Z_o)^{n+1} = 2\left[\frac{2.375(4.77)}{1.4}\right](.23)^{1.4} = 2.1 \text{ LBS.}$$

PROPOSED EQUATION:

$$H = A\left\{c\left[1 + \frac{K}{i_o\ell}\left(e^{\frac{-i_o\ell}{K}} - 1\right)\right] + \left(\frac{k_c}{b} + k_\phi\right) Z_o^n \tan\phi\left[\frac{1}{n+1} - \frac{1}{\ell^{n+1}}\int_0^\ell x^n e^{\frac{-i_o x}{K}}\,dx\right]\right\}$$

$$\text{LET } J = \frac{i_o\ell}{K} \qquad 1 + \frac{K}{i_o\ell}\left(e^{\frac{-i_o\ell}{K}} - 1\right) = 1 - \frac{1}{J}\left(1 - e^{-J}\right)$$

$k = 0.30$
$\ell = 18.0$
$i_o =$ SLIP

SLIP %	$1 - \frac{1}{J}(1 - e^{-J})$
5	.6833
10	.8337
15	.8889
20	.9167
30	.9444
40	.9583
60	.9722
80	.9792
100	.9833

SLIP	$\int_0^{\ell} x^n e^{\frac{-i_o x}{K}}\,dx$ *	(M) $\dfrac{\frac{1}{n+1}\cdot\ell^{n+1}}{\int_0^{\ell} x^n e^{\frac{-i_o x}{K}}\,dx}$	M·N		$\overbrace{c\left[1-\frac{1}{J}(1-e^{-J})\right]}^{O}$	
			MIN.	MAX.	MIN.	MAX.
5	9.652	.5335	1.2744	1.2439	.0715	.2769
10	4.076	.6378	1.5235	1.4870	.0873	.3379
15	2.320	.6706	1.6019	1.5635	.0931	.3603
20	1.545	.6851	1.6365	1.5973	.0960	.3715
30	.8671	.6978	1.6668	1.6269	.0989	.3828
40	.5731	.7033	1.6800	1.6397	.1003	.3884
60	.3165	.7081	1.6914	1.6509	.1018	.3940
80	.2054	.7102	1.6965	1.6558	.1025	.3969
100	.1455	.7113	1.6991	1.6584	.1030	.3985

SLIP	M·N + O		H = A [M·N + O]		DP		$\frac{DP}{W}$	
	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.
5	1.3459	1.5208	115.1	130.0	113.0	127.9	.698	.790
10	1.6108	1.8249	137.7	156.0	135.6	153.9	.837	.950
15	1.6950	1.9238	144.9	164.5	142.8	162.4	.881	1.002
20	1.7325	1.9682	148.1	168.3	146.0	166.2	.901	1.026
30	1.7657	2.0097	151.0	171.8	148.9	169.7	.919	1.048
40	1.780[illegible]	2.0281	152.2	173.4	150.1	171.3	.927	1.057
60	1.79[illegible]	2.0449	153.3	174.8	151.2	172.7	.933	1.066
80	1.7[illegible]90	2.0527	155.8	175.5	151.7	173.4	.936	1.070
100	1.8021	2.0569	154.1	175.9	152.0	173.8	.938	1.073

$$\overbrace{\left(\frac{k_c}{b}+k_\phi\right)z_o^{\,n}\tan\phi}^{N}\quad\begin{cases}\text{MIN.} = 4.77(.556)(.9007) = 2.3887\\ \text{MAX.} = 4.77(.556)(.8791) = 2.3315\end{cases}$$

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AN ANALYSIS OF THE DRAWBAR PULL VS. SLIP RELATIONSHIP FOR TRACK LAYING VEHICLES Zoltan Janosi, Ben Hanamoto

Report No. RR-47, November, 1961 45 pp - Illus - Tables

Project No. 570.05.001 Unclassified Report

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Conclusions are drawn concerning possible means of achieving improved tracked vehicle design, and the direction of future research.

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